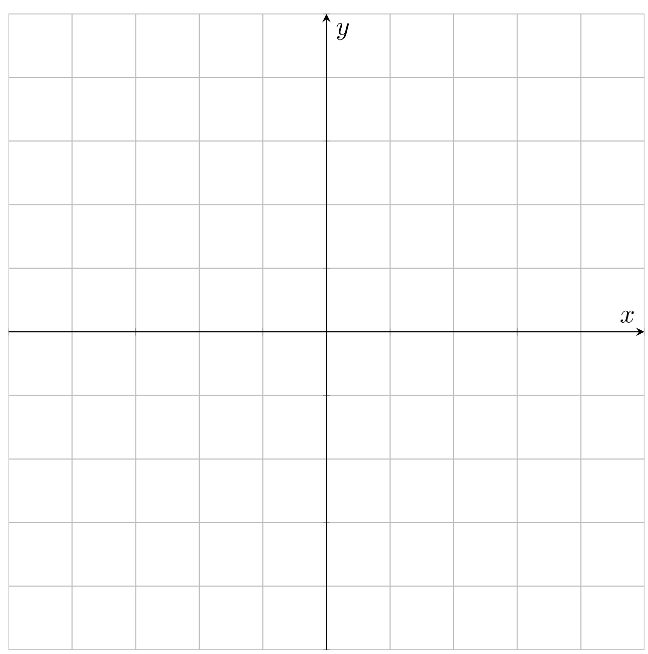
Drone Navigation and Interception

Your platoon is tasked with conducting a surveillance mission using a tactical reconnaissance drone. The drone ascends to a planned altitude of 300 meters and then travels between several checkpoints at a constant speed of 22 m/s during its patrol. Hostile forces may attempt to impede reconnaissance with interception drones.

In this lab you will use vectors to model the reconnaissance drone's motion and determine whether or not an enemy drone will impact at the required kill angle.

Task 1: Plot Drone Initial Position and Check Points

1. Reconnaissance drone checkpoints:
   - SP (0, 0, 0)
   - CP1 (0, 0, 300) *reconnaissance altitude
   - CP2 (500, 200, 300)
   - CP3 (100, 500, 300)
   - CP4 (-500, 300, 300)
   - CP5 (-300, -300, 300)
   - CP6 (0, 0, 300) *return to hover at reconnaissance altitude above RP
   - RP (0, 0, 0)
2. Plot the drone's route after reaching reconnaissance altitude using displacement vectors.



Task 2: Determine if a Hostile Interceptor will Achieve a Kill

Scenario. The reconnaissance drone departs CP5 heading toward CP6 at 22m/s. At that instant, the enemy drone is spotted at $\overrightarrow{r_{E_0}} = \langle 500, -300, 300 \rangle$, flying at 31 m/s on an unknown heading. A kill only occurs if the enemy intercepts the reconnaissance drone before the CP6 *and* impacts orthogonally.

1. **Reconnaissance drone motion.** Build a vector equation modeling the reconnaissance drone's motion at a given time as it flies from CP5 to CP6.

   (a) If the reconnaissance drone's original position vector at CP5 is: $\overrightarrow{r_{R0}} = \langle -300, -300, 300 \rangle$ and the position vector of CP6 is: $\overrightarrow{CP6} = \langle 0, 0, 300 \rangle$, which of the following best represents the displacement vector $\overrightarrow{d} = \overrightarrow{CP5CP6}$?

      i. $\langle 300, 300, 300 \rangle$

      ii. $\langle 300, 300, 0 \rangle$

      iii. $\langle 0, 0, 300 \rangle$

   (b) Using your answer for $\overrightarrow{d}$, calculate the unit vector for displacement, $\hat{d}$, by dividing $\overrightarrow{d}$ by its magnitude.

   (c) Construct the velocity vector for the reconnaissance drone, $\overrightarrow{v_R}$, by multiplying $\hat{d}$ by the speed of the reconnaissance drone in meters per second. *Note: Velocity is a vector, providing the rate of travel (speed) and the direction of travel in one expression. If we find the magnitude of $\overrightarrow{v_R}$, it should return the speed of travel of the drone.*

   (d) Now, using $\overrightarrow{r_{R0}}$ and $\overrightarrow{v_R}$ write the reconnaissance drone's position vector at any time t, where $t \geq 0$ in the following form:

$$\overrightarrow{r_R}(t) = \overrightarrow{r_{R_0}} + \overrightarrow{v_R}t \qquad (1)$$

   Write this equation here:

(e) Compare this equation to the slope-intercept form of a line ($y = mx + b$). How are these equations similar?

2. **Enemy drone motion.** Build a vector equation modeling the enemy drone's motion at a given time as it flies to intercept the friendly drone in the form:

$$\overrightarrow{r_E}(t) = \overrightarrow{r_{E_0}} + \overrightarrow{v_E}t \tag{2}$$

Remember the enemy drones initial position at detection ($t = 0$) is given by the position vector $\overrightarrow{r_{E_0}} = \langle 500, -300, 300 \rangle$. Its velocity vector is unknown. Let $\overrightarrow{v_E} = \langle v_x, v_y, v_z \rangle$. Write the vector equation here:

3. **Collision condition.** Determine if it is possible for the enemy drone to collide with the friendly drone prior to its arrival at CP6 using the given information.

(a) A collision happens when both drones are in the same position at the same time. We will call the time when this occurs t^*. Mathematically we say:

$$\overrightarrow{r_R}(t^*) = \overrightarrow{r_E}(t^*) \tag{3}$$

To determine if a collision is possible, we must solve for the time, t^*, and components of velocity, $\langle v_x, v_y, v_z \rangle$, that make Equation 3 true.

Substitute the equations you found in 1.d and 2 into Equation 3 and write it here:

(b) Break this equation into it's components so that you are left with a system of three equations representing the movement in each direction. Write the three equations:

(c) Using the equations from 3.b above, solve algebraically for v_x, v_y, and v_z.

(d) Now that you have solved for v_x, v_y, and v_z, what information are you still missing to find numerical values for each component?

(e) What given information might could help us formulate a fourth equation to solve for the fourth unknown? *Hint: Think about the relationship between speed and velocity.*

(f) Write the equation to calculate magnitude of the enemy drone's velocity here:

(g) Using the equations from 3.c and 3.e, finish solving the system of equations algebraically for t^* an v_x.

(h) Calculate $\overrightarrow{r_E}(t^*)$. Does the collision occur prior to the reconnaissance drone's arrival at CP6?

4. **Kill-angle requirement.** A kill occurs only if the enemy drone impacts the reconnaissance drone orthogonally. If the interceptor hits at any other angle, it will damage but not destroy the reconnaissance drone.

   (a) What term from the vector equations representing the drones positions, give you the direction of travel?

   (b) How can you use these vectors to determine if impact will be orthogonal?

   (c) Determine if the enemy drone achieves a kill upon collision with the reconnaissance drone. *Hint: Does $\vec{v}_E \cdot \vec{v}_R = 0$?*

5. **Report your results.** Report the following in the Recommendation paragraph of your memorandum:

   (a) the impact point (x, y, z) coordinates,
   (b) the impact time t^* (seconds after detection),
   (c) $\vec{v}_E$ (components in m/s) and its magnitude,
   (d) will the enemy drone achieve a kill prior to arrival at CP6?
   (e) based on your analysis, do you recommend the reconnaissance drone operator change course between CP5 and CP6?

You will only have one enclosure for your memorandum – this worksheet. Ensure you update the memo to reflect.